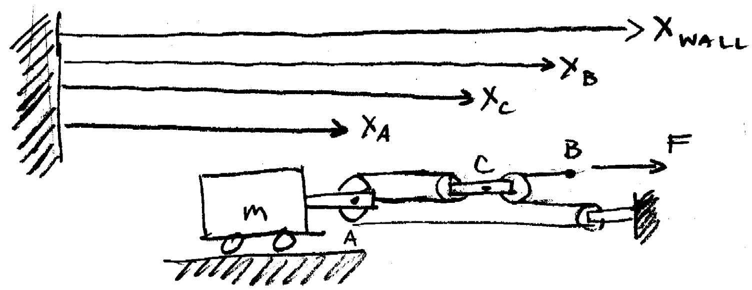
#12.19



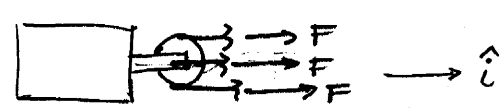
L = length of rope

$$\approx (X_{WALL} - X_A) + 2(X_C - X_A) + (X_{WALL} - X_C) + (X_B - X_C)$$

$$\Rightarrow \; -3\ddot{X}_A + \ddot{X}_B = 0 \quad \Rightarrow \quad \boxed{\ddot{X}_B = 3\ddot{X}_A} \quad (1)$$

Assuming massless pulleys the tension in the rope will be equal to the applied force F.

FBD of mass A



LMB for mass A

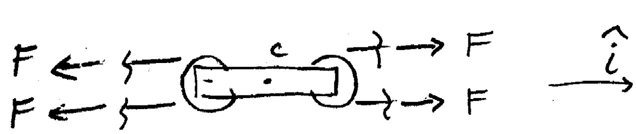
$$\underline{F} = m\underline{a}$$

$$3F\,\hat{i} = m\,\ddot{X}_A\,\hat{i}$$

$$\{\cdot\}\cdot\hat{i} \Longrightarrow \boxed{\ddot{X}_A = 3\frac{F}{m}}$$

$$\overset{(1)}{\Longrightarrow} \boxed{\ddot{X}_B = 9\frac{F}{m}}$$

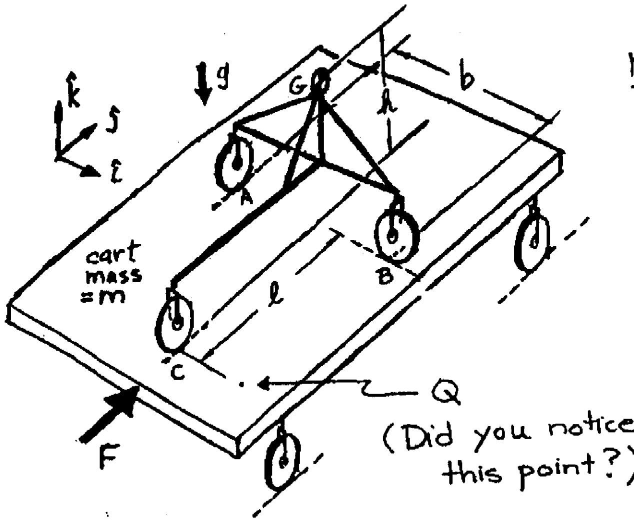
Finally, $\boxed{\ddot{X}_c = 0}$ from the following FBD & LMB



1)(30 pts) **3-wheeled robot.** A 3-wheeled robot with mass m is being transported on a level flatbed trailer also with mass m. The trailer is being pushed with a force $F\hat{\jmath}$. The ideal massless trailer wheels roll without slip. The ideal massless robot wheels also roll without slip. The robot steering mechanism has turned the wheels so that wheels at A and C are free to roll in the $\hat{\jmath}$ direction and the wheel at B is free to roll in the $\hat{\imath}$ direction. The center of mass of the robot at G is h above the trailer bed and symmetrically above the axle connecting wheels A and B. The wheels A and B are a distance b apart. The length of the robot is ℓ.

Find the force vector $\underline{\mathbf{F}}_A$ of the trailer on the robot at A in terms of some or all of $m, g, \ell, F, b, h, \hat{\imath}, \hat{\jmath}$, and $\hat{k}$.

[Hints: Use a free body diagram of the cart with robot to find their acceleration. With reference to a free body diagram of the robot, use angular momentum balance about axis BC to find F_{Az}.]

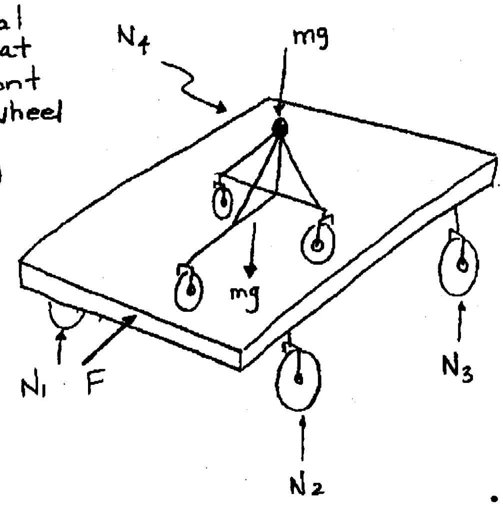


Note: From the announcement,

$$\boxed{\underline{a}_G = \underline{a}_{CART} = \underline{a}}$$

The robot does not move with respect to the cart.

FBD (cart w/robot)



$m_{ROBOT} + m_{CART} = 2m$

LMB : $\underline{\Sigma F} = m_{TOTAL}\, \underline{a}$

LMB $\cdot \hat{\jmath} \Rightarrow F = 2m a_y$

$\therefore \boxed{a_y = \frac{F}{2m}}$

LMB $\cdot \hat{\imath} \Rightarrow 0 = 2m a_x$

$\therefore \boxed{a_x = 0}$

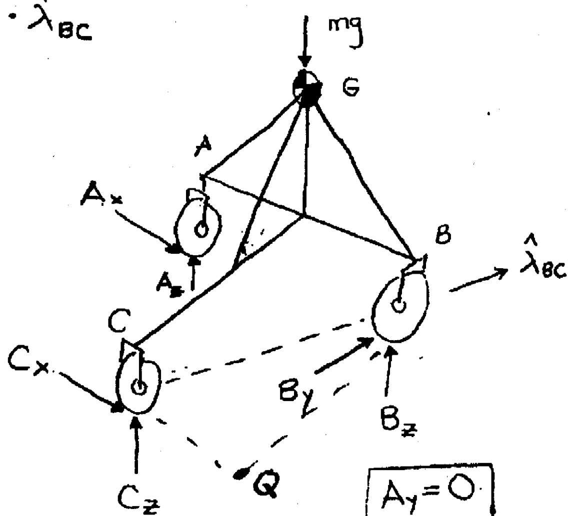
$a_z = 0$ by the assumption that the cart doesn't leave the ground

$\therefore \boxed{\underline{a} = \frac{F}{2m}\hat{\jmath}}$

(Continue work for problem 1 here)

$\underline{AMB}_{/axis\ BC}$: $\{\Sigma \underline{M}_{/c} = \dot{\underline{H}}_{/c}\} \cdot \hat{\lambda}_{BC}$

Where $\hat{\lambda}_{BC} = \dfrac{\underline{r}_{BC}}{|\underline{r}_{BC}|}$



$\Rightarrow \hat{\lambda}_{BC} = \dfrac{\frac{b}{2}\hat{i} + \ell\hat{j}}{\sqrt{(\frac{b}{2})^2 + \ell^2}}$

The only forces creating moments about axis BC are A_z, mg :

$\boxed{A_y = 0}$

↑ wheel rolls freely in $\hat{j}$-dir.

$$\{\Sigma \underline{M}_{/c}\} \cdot \hat{\lambda}_{BC} = \{\underline{r}_{A/c} \times A_z\hat{k} + \underline{r}_{G/c} \times -mg\hat{k}\} \cdot \hat{\lambda}_{BC}$$

$$= \{(-\tfrac{b}{2}\hat{i} + \ell\hat{j}) \times A_z\hat{k} + (\ell\hat{j} + h\hat{k}) \times -mg\hat{k}\} \cdot \hat{\lambda}_{BC}$$

$$= \{+A_z\tfrac{b}{2}\hat{j} + A_z\ell\hat{i} - mg\ell\hat{i}\} \cdot \frac{\frac{b}{2}\hat{i} + \ell\hat{j}}{\sqrt{(\frac{b}{2})^2 + \ell^2}}$$

$$= \left(\tfrac{b}{2}\ell(A_z - mg) + \tfrac{b\ell}{2}A_z\right)\frac{1}{\sqrt{(\frac{b}{2})^2 + \ell^2}}$$

$$\{\dot{\underline{H}}_{/c}\} \cdot \hat{\lambda}_{BC} = \{\underline{r}_{G/c} \times m\underline{a}\} \cdot \hat{\lambda}_{BC}$$

$$= \{(\ell\hat{j} + h\hat{k}) \times m(\tfrac{F}{2m}\hat{j})\} \cdot \hat{\lambda}_{BC}$$

$$= \{-F\tfrac{h}{2}\hat{i}\} \cdot \frac{\frac{b}{2}\hat{i} + \ell\hat{j}}{\sqrt{(\frac{b}{2})^2 + \ell^2}}$$

$$= -\frac{Fbh}{4} \cdot \frac{1}{\sqrt{(\frac{b}{2})^2 + \ell^2}}$$

$$\Rightarrow \{\Sigma \underline{M}_{/c} = \dot{\underline{H}}_{/c}\} \cdot \hat{\lambda}_{BC}$$

$$\frac{b\ell}{2}(A_z - mg) + \frac{b\ell}{2}A_z = -\frac{Fbh}{4}$$

$$b\ell A_z = \frac{mgb\ell}{2} - \frac{Fhb}{4}$$

$$\therefore \boxed{A_z = \frac{mg}{2} - \frac{Fh}{4\ell}}$$

• Now get A_x by taking $AMB_{/Q} \cdot \hat{k}$

$\underline{AMB}_{/Q}$: $\{\Sigma \underline{M}_{/Q} = \dot{\underline{H}}_{/Q}\} \cdot \hat{k}$

The only force creating a moment about Q in the $\hat{k}$-direction is A_x !

$$\Rightarrow \Sigma \underline{M}_{/Q} \cdot \hat{k} = \{\underline{r}_{A/Q} \times A_x\hat{i}\} \cdot \hat{k}$$

$$= \{(-b\hat{i} + \ell\hat{j}) \times A_x\hat{i}\} \cdot \hat{k}$$

$$= \{-A_x\ell\hat{k}\} \cdot \hat{k} = -A_x\ell$$

$$\dot{\underline{H}}_{/Q} \cdot \hat{k} = \{\underline{r}_{G/Q} \times m\underline{a}\} \cdot \hat{k}$$

$$= \{(-\tfrac{b}{2}\hat{i} + \ell\hat{j} + h\hat{k}) \times m(\tfrac{F}{2m}\hat{j})\} \cdot \hat{k}$$

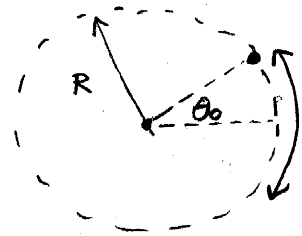
$$= \{\tfrac{F}{2}(-\tfrac{b}{2}\hat{k} - h\hat{i})\} \cdot \hat{k}$$

$$= -\frac{Fb}{4}$$

$$\Rightarrow \{\Sigma \underline{M}_{/Q} = \dot{\underline{H}}_{/Q}\} \cdot \hat{k}$$

$$-A_x\ell = \frac{-Fb}{4} \Rightarrow \boxed{A_x = \frac{Fb}{4\ell}}$$

13.13



$\theta(t) = \theta_0 \cos \lambda t$

The acceleration of a particle moving in circular motion w/ given $\Theta(t)$ is

$$\underline{a}(t) = -R\dot{\theta}^2\,\hat{e}_r + R\ddot{\theta}\,\hat{e}_\theta$$

$$= -R\left[-\lambda\theta_0 \sin \lambda t\right]^2 \hat{e}_r - R\lambda^2\theta_0 \cos \lambda t\ \hat{e}_\theta$$

$$= -R\lambda^2\left[\theta_0^2 - \theta_0^2\cos^2 \lambda t\right]\hat{e}_r - R\lambda^2\,\theta_0 \cos \lambda t\ \hat{e}_\theta$$

$$\Rightarrow \quad \underline{a}(\theta(t)) = -R\lambda^2(\theta_0^2 - \theta^2)\,\hat{e}_r - R\lambda^2\theta\,\hat{e}_\theta$$

where $\theta \in [-\theta_0, \theta_0]$. Thus

$$\|\underline{a}\| = \left(R^2\lambda^4\left[\theta_0^2 - \theta^2\right]^2 + R^2\lambda^4\theta^2\right)^{1/2}$$

$$= \left(R^2\lambda^4\theta_0^4 - 2R^2\lambda^4\theta_0^2\theta^2 + R^2\lambda^4\theta^4 + R^2\lambda^4\theta^2\right)^{1/2}$$

From Calculus, $\|\underline{a}\|$ is extremized when either $\theta = \pm\theta_0$ or

$$\left.\frac{d\|\underline{a}\|}{d\theta}\right|_{\theta=\theta_E} = 0$$

$$= \frac{1}{2}\left[\quad\right]^{-\frac{1}{2}}\left(-4R^2\lambda^4\theta_0^2\theta_E + 4R^2\lambda^4\theta_E^3 + 2R^2\lambda^4\theta_E\right)$$

Thus θ_E must satisfy

$$\theta_E\left[-4R^2\lambda^4\theta_0^2 + 2R^2\lambda^4 + 4R^2\lambda^4\theta_E^2\right] = 0$$

$$\Rightarrow \quad \theta_E = 0,\ \boxed{\pm\sqrt{\theta_0^2 - \frac{1}{2}}}$$

These are local minima for $\theta_0^2 > \frac{1}{2}$ so we ignore them...

For $\theta_E = 0$ to be a local maximum it must satisfy

$$\left.\frac{d^2 \|\underline{a}\|}{d\theta^2}\right|_{\theta=0} < 0$$

This derivative is a little messy, but when we plug in $\theta = 0$ we get

$$\left.\frac{d^2 \|\underline{a}\|}{d\theta^2}\right|_{\theta=0} = \frac{(1-2\theta_0^2)R\lambda^2}{\theta_0^2} <$$

$$\Rightarrow \boxed{\theta_0^2 > \frac{1}{2}}$$

UNNECESSARY

Finally, for $\theta = 0$ to be a global maximum we must compare $\|\underline{a}(0)\|$ & $\|\underline{a}(\pm\theta_0)\|$.

$$\|\underline{a}(0)\| = \theta_0^2 R\lambda^2$$

$$\|\underline{a}(\pm\theta_0)\| = |\theta_0| R\lambda$$

Thus $\|\underline{a}(0)\| > \|\underline{a}(\pm\theta_0)\|$ when $|\theta_0| > 1$.

$$\boxed{\text{Therefore } \theta = 0 \text{ maximizes } \|\underline{a}\| \text{ when } |\theta_0| > 1.}$$